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Determination of Chaotic Attractors in the Rat Brain

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The existence of low-dimensional deterministic structures in experimental time series, derived from the occurrences of spikes in electrophysiological recordings from rat brains, has been revealed in 7 out of 27 samples. The correlation dimension of the chaotic attractors ranged between 0.14 and 3.3 embedded in a space of dimension 2–6. A test on surrogate data was also performed.

KEY WORDS: Chaos; nonlinear dynamics; correlation dimension; entropy; spike-trains; electrophysiology.

1. INTRODUCTION

We investigate the existence of chaotic attractors in discrete time series derived from the epochs of action potentials (*spike trains*) recorded extracellularly in the central nervous system.

Several authors have considered the problem of the existence of a dynamical structure in a time series (see refs. 6, 7, and 10 and references therein), though questionable results have been produced. Interesting applications to the nervous system can be found in refs. 4 and 8. We apply a numerical method due to Grassberger and Procaccia(7) which allows one to determine the dimension of the phase space and that of the attractor. Due to the limited amount of data in the discrete time series, we devoted particular care to the choice of the parameters, without relying on too optimistic estimations. The current study indicates that most cases (20/27) were characterized by a *white-noise* dynamic behavior. Such white-noise

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dynamics was further confirmed by application of the method to a series of surrogate data. In the remaining experimental cases (7/27) a deterministic structure was observed. The embedding space, and therefore the attractor, were characterized by a low dimension ranging from 2 to 6. We refer to ref. 5 for complete details.

2. EXPERIMENTAL DATA

The data set included 27 single-unit spike trains, recorded with an accuracy of 1 msec, in the substantia nigra pars reticulata ($n = 13$) and auditory thalamus ($n = 14$) of five anesthetized young adult Long-Evans rats.(13) The recordings were performed using extracellular microelectrodes with an impedence in the range 0.5–2 MΩ at a frequency of 1 kHz. The same microelectrode can record up to four distinct single units (we refer to ref. 12 for complete details). The data were collected during *spontaneous activity* (i.e., without external stimulation) and acoustically evoked stimulation (i.e., binaural *white-noise* bursts lasting 200 msec at 20 dB above threshold). In order to avoid spurious detection of chaotic dynamics in the experimental data, we tested our analysis on surrogate data, as suggested in ref. 9. More specifically, we analyzed seven simulated data sets produced according to ref. 1. The simulations corresponded to 200 sec of simulated time with ten simultaneous spike trains for each computer-generated file. Two data sets were realizations of independent and stationary Poisson processes at different firing rates, while two more data sets were derived from nonstationary Poisson processes. The last three cases were realizations of correlated and nonstationary Poisson processes at different firing rates and different rates of fluctuation.

The time series is defined by the time intervals between two successive spikes, say $\{x_1,..., x_K\} \in \mathbf{Z}^K$ (where K is the total number of recordings). We analyzed these time series using techniques of dynamical system theory.

3. DATA ANALYSIS

Several mathematical algorithms have been developed to investigate the phase-space structure associated with experimental data.(6, 7, 10) One of the most widely accepted methods is due to Grassberger and Procaccia,(7) which allows one to determine the dimension of the embedding phase space as well as the existence of an attractor. We briefly review their technique.

Let $\{x_1,..., x_K\}$ be the experimental time series; in a d-dimensional embedding space we define *delay coordinates* by setting

$$\begin{aligned} y_1 &= (x_1, \ldots, x_d) \\ y_2 &= (x_2, \ldots, x_{d+1}) \\ &\ldots \\ y_N &= (x_N, \ldots, x_K) \end{aligned} \tag{1}$$

where $N = K - d + 1$. The embedding space is therefore the set of points $\{y_j \in \mathbf{Z}^d\}_{j=1,\ldots,N}$. For a positive $r > 0$, let $n_j(r; d)$ be the number of points whose distance from y_j is less than r. Denoting by Θ the Heaviside function, one has

$$n_j(r; d) = \sum_{i=1, i \neq j}^{N} \Theta(r - |y_i - y_j|_d) \tag{2}$$

where $|\cdot|_d$ is the standard Euclidean norm in $\mathbf{R}^d$. An average of the above quantities is provided by the *correlation integral*, defined as

$$C_{N,d}(r) = \frac{1}{N^2} \sum_{j=1}^{N} n_j(r; d) \tag{3}$$

Grassberger and Procaccia showed that in the limit of small r, $C_{N,d}(r)$ scales as r^δ, where δ is the so-called *correlation dimension*, which measures the size of the attractor. More precisely,

$$\delta \equiv \lim_{r \to 0} \lim_{N \to \infty} \frac{d \log C_{N,d}(r)}{d \log r} \tag{4}$$

In practical applications, in order to determine the correlation dimension, we plot the graph of $\log C_{N,d}(r)$ vs. $\log r$, for several values of d and large values of r. If the phase space has a deterministic structure, the slope of the above curves becomes nearly constant as the embedding dimension is varied, determining the value of the correlation dimension. A white-noise signal can be recognized by a linear increase of the slope with d.

The slope of the curves has to be determined in a particular region, usually called the *scaling region*. More precisely, for small values of r, say $r < r_0$, few points fall in a neighborhood of radius r and the curve $\log C_{N,d}(r)$ vs. $\log r$ is very irregular. On the other hand, for big values, say $r > r_1$, the curve tends to flatten. Therefore the slope of the curve can be computed in the scaling region (r_0, r_1).

A *local singular value decomposition* (SVD)(3) can be applied to lessen the experimental noise. In the d-dimensional embedding space, let $y_0^{(1)}, \ldots, y_0^{(N_c)}$ (for some $N_c > 0$) be arbitrary local centers. For each of these centers, we determine the ν closest points, say $(y_1^{(\alpha)}, ..., y_\nu^{(\alpha)})$, such that the union of the sets $\{y_0^{(\alpha)}, y_1^{(\alpha)}, ..., y_\nu^{(\alpha)}\}$, $\alpha = 1, ..., N_c$, is a pairwise disjoint covering of the original data set. We decompose each of the $(\nu + 1) \times d$ matrices

$$E^{(\alpha)} \equiv (y_0^{(\alpha)} y_1^{(\alpha)} \cdots y_\nu^{(\alpha)})^T, \qquad \alpha = 1, ..., N_c \tag{5}$$

as

$$E^{(\alpha)}_{(\nu+1)\times d} = W^{(\alpha)}_{(\nu+1)\times d} \cdot S^{(\alpha)}_{d\times d} \cdot (V^{(\alpha)}_{d\times d})^T \tag{6}$$

where $W^{(\alpha)}$, $V^{(\alpha)}$ are orthogonal matrices and $S^{(\alpha)}$ is diagonal with entries given by the *singular values* $\{s_j^{(\alpha)}\}$ associated to $E^{(\alpha)}$. If some of these values are dominant, the projection of the phase space on the corresponding directions provides the essential information on the dynamical structure. A spatiotemporal measure is provided by the entropy σ, which can be computed in terms of the correlation function as

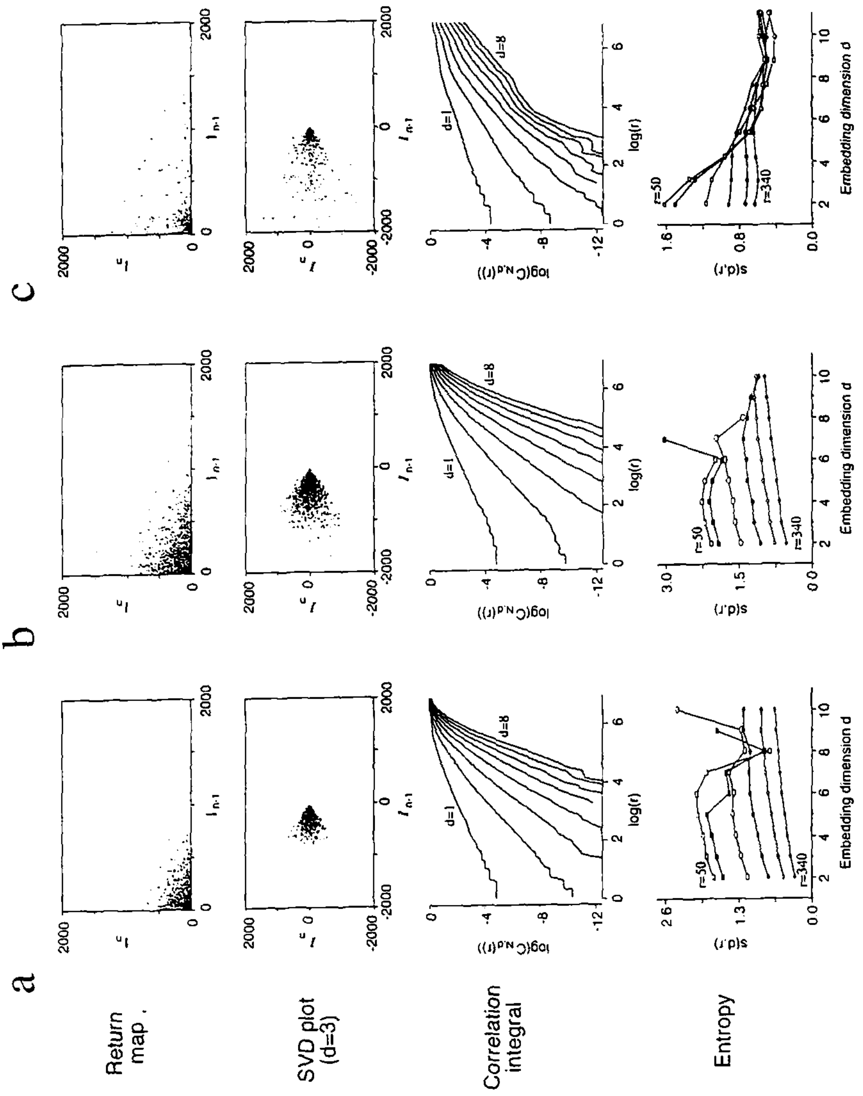
$$\sigma \equiv \sigma(d, r) \sim \log \frac{C_{N,d}(r)}{C_{N,d+1}(r)} \tag{7}$$

in the limit of large embedding dimensions and for different values of r. It can be shown that the entropy is less than or equal to the sum of the positive Lyapunov exponents.

4. RESULTS AND CONCLUSIONS

We found evidence of the existence of chaotic attractors in 7 out of 27 cases. The deterministic samples contained on the average 1458 events and a minimum of 854 events. Four cases were found during spontaneous activity (two for each investigated region). The remaining three cases were observed during the acoustically evoked condition in the auditory thalamus. In all cases the embedding dimension ranged between 2 and 6, while the correlation dimension varied between 0.14 and 3.3. The entropy was estimated between 0.40 and 1.25.

Fig. 1. (a) Simulated data: no chaotic dynamics. (b) A stochastic sample recorded in the substantia nigra pars reticulata during spontaneous activity: no chaotic dynamics. (c) A deterministic sample recorded in the auditory thalamus during spontaneous activity. The dynamical system is characterized by an embedding space of dimension three and a chaotic attractor of dimension 1.8. Note that the SVD plot is performed for an embedding dimension $d = 3$. The correlation integrals are computed for embedding dimensions $d = 1$ to $d = 8$. The entropy is computed for $r = 50$, 60, 100, 160, 240, 340.


a
b
c
Return map
SVD plot (d=3)
Correlation integral
Entropy
I_n
I_{n-1}
2000
0
-2000
log(C_{N,d}(r))
log(r)
d=1
d=8
s(d,r)
Embedding dimension d
r=50
r=340
2.6
1.3
3.0
1.5
1.6
0.8
0.0

We report in Fig. 1 the return map, the result of the application of the local singular value decomposition, the computation of the correlation integral, and the estimate of the entropy for three samples. Figure 1a refers to a surrogate data set, obtained according to ref. 1 and containing 1028 points. All 70 simulated spike trains showed the same dynamics as Fig. 1a. The random behavior is indicated by the linear increase of the slopes of the correlation integral functions. A similar random dynamical process is shown in Fig. 1b, which reports the analysis of a sample (containing 2003 points) recorded in the substantia nigra pars reticulata during spontaneous activity. An example of a deterministic sample is provided in Fig. 1c, related to a time series containing 1027 events recorded in the auditory thalamus during spontaneous activity. The convergence of the slopes of $\log C_{N,d}(r)$ versus $\log r$ suggests the existence of a chaotic attractor with correlation dimension 1.8 in a three-dimensional embedding space. In this case the entropy seems to converge around the value $\sigma = 0.50$. Note that SVD plots may be confusing and would lead to misinterpretations if no other tests for nonlinearity are performed.

This study has provided the first evidences of low-dimensional chaotic behavior in single unit spike trains. This suggests that it may exist in a number of neuronal networks, each one being potentially described by a set of differential equations, which may interact at the level of selected single units. Therefore, the analysis of chaotic attractors might provide a new measure of the level of interacting networks at different conditions, encompassing also clinical and pharmacological manipulations. In addition, it may provide a tool to compare the activity across regions recorded in the same condition. Time-related neuronal coding has recently been a matter of interest, especially at the level of the cerebral cortex.(2, 11) It is certainly necessary to extend our analysis to such brain regions in order to determine if chaotic dynamics may coexist with precise timed structures in the spike trains.

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